3. $\frac{1}{2} t\, e^{-3\,t}$

```
ClearAll["Global`*"]
```

$$e1 = \text{LaplaceTransform}\left[\frac{1}{2} t\, e^{-3\,t}, t, s\right]$$

$$\frac{1}{2\,(3 + s)^2}$$

Above: The answer matches the text.

5. $t\, \text{Cos}[\omega\, t]$

```
ClearAll["Global`*"]
```

$$e1 = \text{LaplaceTransform}[t\, \text{Cos}[\omega\, t], t, s]$$

$$\frac{s^2 - \omega^2}{\left(s^2 + \omega^2\right)^2}$$

Above: The answer matches the text.

7. $t^2\, \text{Cosh}[2\,t]$

```
ClearAll["Global`*"]
```

$$\text{LaplaceTransform}\left[t^2\, \text{Cosh}[2\,t], t, s\right]$$

$$\frac{2\,s\,\left(12 + s^2\right)}{\left(-4 + s^2\right)^3}$$

Above: The answer matches the text.

9. $\frac{1}{2} t^2\, \text{Sin}[\pi\, t]$

```
ClearAll["Global`*"]
```

$$\text{e1 = LaplaceTransform}\left[\frac{1}{2} t^2 \, \text{Sin}[\pi t], t, s\right]$$

$$-\frac{\pi \left(\pi^2 - 3 \, s^2\right)}{\left(\pi^2 + s^2\right)^3}$$

Above: The answer matches the text.

11. $4 \, t \, \text{Cos}\left[\frac{1}{2} \pi t\right]$

ClearAll["Global`*"]

$$\text{e1 = LaplaceTransform}\left[4 \, t \, \text{Cos}\left[\frac{1}{2} \pi t\right], t, s\right]$$

$$\frac{16 \left(-\pi^2 + 4 \, s^2\right)}{\left(\pi^2 + 4 \, s^2\right)^2}$$

$$\text{e2} = \frac{\left(-\pi^2 + 4 \, s^2\right)}{\left(\frac{1}{4} \pi^2 + s^2\right)^2}$$

$$\frac{-\pi^2 + 4 \, s^2}{\left(\frac{\pi^2}{4} + s^2\right)^2}$$

e3 = FullSimplify[e1 == e2]
True

Above: The answer matches the text.

14 - 20 Inverse transforms
Using differentiation, integration, s-shifting, or convolution, and showing the details, find f[t] if $\mathcal{L}[f]$ equals:

15. $\dfrac{s}{\left(s^2 - 9\right)^2}$

ClearAll["Global`*"]

$$\text{e1 = InverseLaplaceTransform}\left[\frac{s}{\left(s^2 - 9\right)^2}, s, t\right]$$

$$\frac{1}{12} \, e^{-3 \, t} \left(-1 + e^{6 \, t}\right) t$$

e2 = ExpToTrig[e1]

$\frac{1}{12}$ t (Cosh[3 t] – Sinh[3 t]) (– 1 + Cosh[6 t] + Sinh[6 t])

e3 = FullSimplify[e2]

$\frac{1}{6}$ t Sinh[3 t]

Above: The answer matches the text.

17. Log$\left[\frac{s}{s-1}\right]$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\text{Log}\left[\frac{s}{s-1}\right], s, t\right]$

$-\frac{1-e^t}{t}$

Above: The answer matches the text.

19. Log$\left[\frac{s^2+1}{(s-1)^2}\right]$

ClearAll["Global`*"]

e1 = InverseLaplaceTransform$\left[\text{Log}\left[\frac{s^2+1}{(s-1)^2}\right], s, t\right]$

$\frac{2\left(e^t - \text{Cos}[t]\right)}{t}$

Above: The answer matches the text.